

Section 2

CSCI E-22

Will Begin Shortly

Practice with Recursion

The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

```
public static String removeCapitals(String s) {  
    if (s == null)  
        throw new IllegalArgumentException();  
  
    if (s.equals(""))  
        return "";  
  
    String removedFromRest = _____;  
  
    ...  
  
}
```

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        throw new IllegalArgumentException();  
  
    if (s.equals(""))  
        return "";  
  
    String removedFromRest = _____;  
  
    ...  
  
}
```

removeCapitals("HeLlO")



return "eo"

Recall: Recursive Problem Solving

When solving problems using recursion, we break the problem down into smaller subproblems.

Once we have broken down the problem into the smallest subproblem (one that we can solve), we have reached a base case.

Then, we can progressively build up the solutions to the subproblems until we have a solution for the overall problem.

$$\text{sum}(n) =$$

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$$\text{sum}(n) = n + \text{sum}(n-1)$$

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Then, we can progressively build up the solutions to the subproblems until we have a solution for the overall problem.

$$\text{sum}(n) = n + \text{sum}(n-1)$$

```
public static int sum(int n) {  
    if (n <= 0)  
        return 0;  
  
    int rest = sum(n - 1);  
    return n + rest;  
}
```

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$$\text{sum}(n) = n + \text{sum}(n-1)$$

```
public static int sum(int n) {  
    if (n <= 0)  
        return 0;  
  
    int rest = sum(n - 1);  
    return n + rest;  
}
```

```
sum(4) = 4 + sum(3)  
sum(3) = 3 + sum(2)  
sum(2) = 2 + sum(1)  
sum(1) = 1 + sum(0)  
sum(0) = 0
```

Recall: Recursive Problem Solving

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Once we have broken down the problem into the smallest subproblem (one that we can solve), we have reached a base case.

Then, we can progressively build up the solutions to the subproblems until we have a solution for the overall problem.

$$\text{sum}(n) = n + \text{sum}(n-1)$$

```
public static int sum(int n) {  
    if (n <= 0)  
        return 0;  
  
    int rest = sum(n - 1);  
    return n + rest;  
}
```

```
sum(4) = 4 + sum(3) }  
sum(3) = 3 + sum(2) }  
sum(2) = 2 + sum(1) }  
sum(1) = 1 + sum(0) }  
sum(0) = 0
```

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
- What's the recursive subproblem?
- What work do we need to do before returning?

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The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

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Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
 - When `s` is the empty string
- What's the recursive subproblem?
- What work do we need to do before returning?

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
 - When `s` is the empty string
- What's the recursive subproblem?
 - Removing the capitals from the rest of the string
- What work do we need to do before returning?

```
removeCapitals("HeLLo")
```

If we knew the solution to `removeCapitals("eLLo")`, we could solve `removeCapitals("HeLLo")`

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
 - When `s` is the empty string
- What's the recursive subproblem?
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- What work do we need to do before returning?

```
removeCapitals("HeLLo") = H? + removeCapitals("eLLo")
```

If we knew the solution to `removeCapitals("eLLo")`, we could solve `removeCapitals("HeLLo")`

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
 - When s is the empty string
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`removeCapitals("HeLLo") = H? + removeCapitals("eLLo")`

If we knew the solution to `removeCapitals("eLLo")`, we could solve `removeCapitals("HeLLo")`

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
 - When s is the empty string
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 - Removing the capitals from the rest of the string
- What work do we need to do before returning?
 - Test whether the first character in the string is a capital; if it is, it should not be included in the return value

`removeCapitals("HeLLo") = H? + removeCapitals("eLLo")`

If we knew the solution to `removeCapitals("eLLo")`, we could solve `removeCapitals("HeLLo")`

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

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`removeCapitals("HeLLo") = H + removeCapitals("eLLo")`

If we knew the solution to `removeCapitals("eLLo")`, we could solve `removeCapitals("HeLLo")`

Recursive Problem Solving Approach

Before writing a recursive method, we can try to plan:

- What's the base case?
 - When s is the empty string
- What's the recursive subproblem?
 - Removing the capitals from the rest of the string
- What work do we need to do before returning?
 - Test whether the first character in the string is a capital; if it is, it should not be included in the return value

`removeCapitals("HeLLo") = H + removeCapitals("eLLo") = "eo"`

If we knew the solution to `removeCapitals("eLLo")`, we could solve `removeCapitals("HeLLo")`

Practice with Recursion

The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

```
public static String removeCapitals(String s) {  
    if (s == null)  
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    String removedFromRest = _____;  
  
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    if (s.equals(""))  
        return "";  
  
    String removedFromRest = removeCapitals(s.substring(1));  
  
    char first = s.charAt(0);  
    if (_____)  
  
}
```

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        return "";  
  
    String removedFromRest = removeCapitals(s.substring(1));  
  
    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
        return _____;  
  
}
```

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```
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    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return _____;  
}
```

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    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
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    else  
        return first + removedFromRest;  
}
```

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    String removedFromRest = removeCapitals(s.substring(1));  
  
    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

removeCapitals	
s	<input type="text" value="HeLlO"/>
removedFromRest	<input type="text"/>

Practice with Recursion

The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

```
public static String removeCapitals(String s) {  
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        return "";  
  
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    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

removeCapitals	
s	<input type="text" value="eLlO"/>
removedFromRest	<input type="text"/>

removeCapitals	
s	<input type="text" value="HeLlO"/>
removedFromRest	<input type="text"/>

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The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

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        return "";  
  
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    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

removeCapitals	s	"Llo"
	removedFromRest	

removeCapitals	s	"eLlo"
	removedFromRest	

removeCapitals	s	"HeLlo"
	removedFromRest	

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The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

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    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

removeCapitals	s	"Lo"
	removedFromRest	

removeCapitals	s	"Llo"
	removedFromRest	

removeCapitals	s	"eLlo"
	removedFromRest	

removeCapitals	s	"HeLlo"
	removedFromRest	

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    char first = s.charAt(0);  
    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

removeCapitals	s	"o"
	removedFromRest	
removeCapitals	s	"Lo"
	removedFromRest	
removeCapitals	s	"LLo"
	removedFromRest	
removeCapitals	s	"eLLo"
	removedFromRest	
removeCapitals	s	"HeLLo"
	removedFromRest	

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    else  
        return first + removedFromRest;  
}
```

removeCapitals	s	" "
	removedFromRest	
removeCapitals	s	"o"
	removedFromRest	
removeCapitals	s	"Lo"
	removedFromRest	
removeCapitals	s	"LLo"
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	removedFromRest	
removeCapitals	s	"HeLLo"
	removedFromRest	

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        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

removeCapitals	s	"o"
	removedFromRest	" "

removeCapitals	s	"Lo"
	removedFromRest	

removeCapitals	s	"LLo"
	removedFromRest	

removeCapitals	s	"eLLo"
	removedFromRest	

removeCapitals	s	"HeLLo"
	removedFromRest	

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        return first + removedFromRest;  
}
```

removeCapitals	s	"Lo"
	removedFromRest	"o"

removeCapitals	s	"LLo"
	removedFromRest	

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}
```

removeCapitals	
s	<input type="text" value="LLo"/>
removedFromRest	<input type="text" value="o"/>

removeCapitals	
s	<input type="text" value="eLLo"/>
removedFromRest	<input type="text"/>

removeCapitals	
s	<input type="text" value="HeLLo"/>
removedFromRest	<input type="text"/>

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    else  
        return first + removedFromRest;  
}
```

removeCapitals	
s	<input type="text" value="eLLo"/>
removedFromRest	<input type="text" value="o"/>

removeCapitals	
s	<input type="text" value="HeLLo"/>
removedFromRest	<input type="text"/>

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```

removeCapitals	
s	"HeLlO"
removedFromRest	"eo"

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        return removedFromRest;  
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        return first + removedFromRest;  
}
```

How many calls of this method are needed when...

n is 0?

Practice with Recursion

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        return first + removedFromRest;  
}
```

How many calls of this method are needed when...

n is 0? 1

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        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

How many calls of this method are needed when...

n is 0? 1

n is 1?

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    if (first >= 'A' && first <= 'Z')  
        return removedFromRest;  
    else  
        return first + removedFromRest;  
}
```

How many calls of this method are needed when...

n is 0? 1
n is 1? 2

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n is 0? 1
n is 1? 2
n is 2? 3
n is 3? 4

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}
```

How many calls of this method are needed when...

n is 0? 1
n is 1? 2
n is 2? 3
n is 3? 4

What is the general formula for the number of calls needed to process a string of length n?

Practice with Recursion

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        return first + removedFromRest;  
}
```

How many calls of this method are needed when...

n is 0? 1
n is 1? 2
n is 2? 3
n is 3? 4

What is the general formula for the number of calls needed to process a string of length n?

$n + 1$

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The following Java method should use recursion to remove all capital letters from a string. Some of the code has been omitted and we will need to fill it in:

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How many calls of this method are needed when...

n is 0? 1
n is 1? 2
n is 2? 3
n is 3? 4

What is the general formula for the number of calls needed to process a string of length n?

$n + 1$

Let's remember this while we complete the next exercise

The Fibonacci Sequence

The Fibonacci sequence is a well-known number series in which each number in the series is the sum of the two previous numbers.

We define the first two numbers as $F_0 = 0$ and $F_1 = 1$, and all successive numbers as:

$$F_n = F_{n-1} + F_{n-2}$$

Since the sequence is defined recursively, using recursion to calculate Fibonacci numbers is natural. Finish the code below to write an algorithm that calculates the n th Fibonacci number.

```
public static long fib(int n) {  
    // base case  
    if (_____)   
  
  
  
}
```

The Fibonacci Sequence

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We define the first two numbers as $F_0 = 0$ and $F_1 = 1$, and all successive numbers as:

$$F_n = F_{n-1} + F_{n-2}$$

Since the sequence is defined recursively, using recursion to calculate Fibonacci numbers is natural. Finish the code below to write an algorithm that calculates the n th Fibonacci number.

```
public static long fib(int n) {  
    // base case  
    if (n == 0 || n == 1)  
        return _____;  
  
}
```

The Fibonacci Sequence

The Fibonacci sequence is a well-known number series in which each number in the series is the sum of the two previous numbers.

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We define the first two numbers as $F_0 = 0$ and $F_1 = 1$, and all successive numbers as:

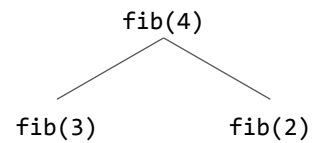
$$F_n = F_{n-1} + F_{n-2}$$

Since the sequence is defined recursively, using recursion to calculate Fibonacci numbers is natural. Finish the code below to write an algorithm that calculates the n th Fibonacci number.

```
public static long fib(int n) {  
    // base case  
    if (n == 0 || n == 1)  
        return n;  
  
    // recursive case  
    return fib(n - 1) + fib(n - 2);  
}
```

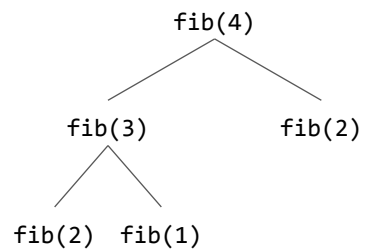

The Fibonacci Sequence

Draw a diagram that shows the number of times `fib()` is called with an initial value of 4. It has been started for you:



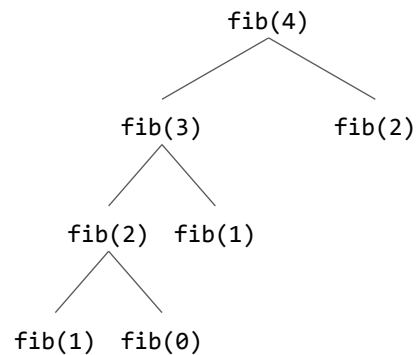
The Fibonacci Sequence

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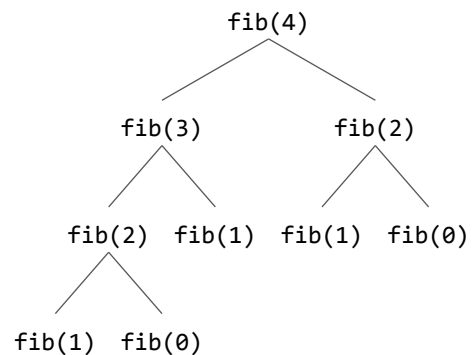
The Fibonacci Sequence

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The Fibonacci Sequence

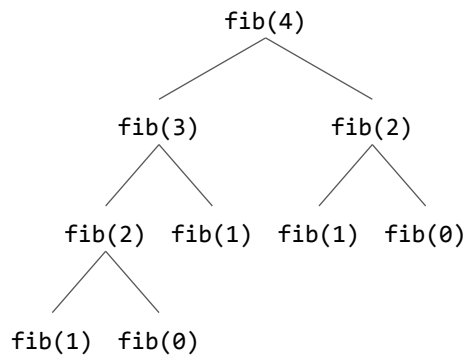
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The Fibonacci Sequence

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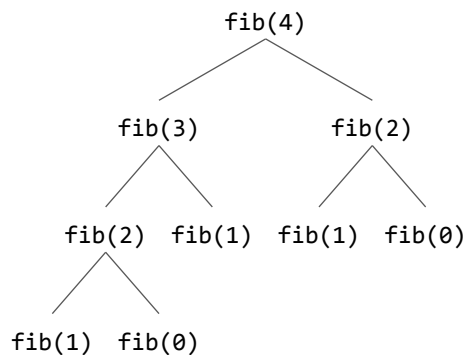
How many times did we call `fib()` to find the fourth Fibonacci number?



The Fibonacci Sequence

Draw a diagram that shows the number of times `fib()` is called with an initial value of 4. It has been started for you:

How many times did we call `fib()` to find the fourth Fibonacci number? 9

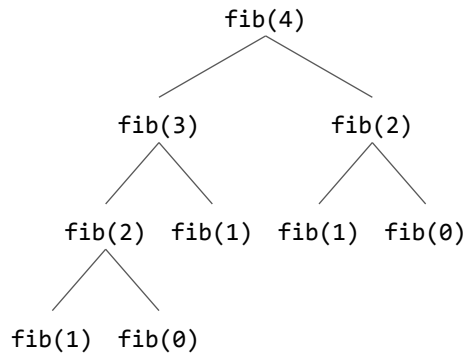


The Fibonacci Sequence

Draw a diagram that shows the number of times `fib()` is called with an initial value of 4. It has been started for you:

How many times did we call `fib()` to find the fourth Fibonacci number? 9

Do you see a problem with this? What if we tried larger numbers, like 50?



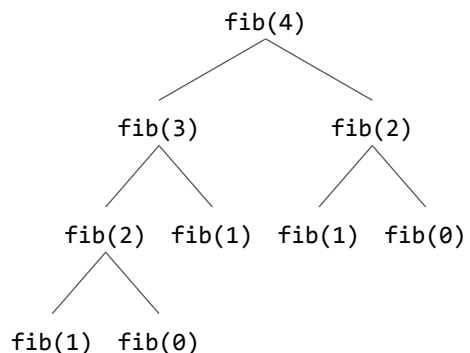
The Fibonacci Sequence

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How many times did we call `fib()` to find the fourth Fibonacci number? 9

Do you see a problem with this? What if we tried larger numbers, like 50?

- The number of calls increases exponentially for the initial value of n . This means that we would have to make about 10 billion method calls to calculate `fib(50)`!



The Fibonacci Sequence

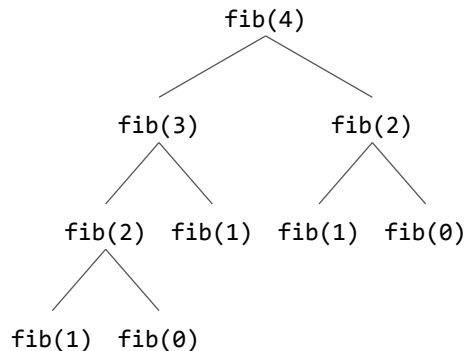
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How many times did we call `fib()` to find the fourth Fibonacci number? 9

Do you see a problem with this? What if we tried larger numbers, like 50?

- The number of calls increases exponentially for the initial value of n . This means that we would have to make about 10 billion method calls to calculate `fib(50)`!

How would you rewrite `fib()` to be more efficient, either still as a recursive method or iteratively?



The Fibonacci Sequence

Draw a diagram that shows the number of times `fib()` is called with an initial value of 4. It has been started for you:

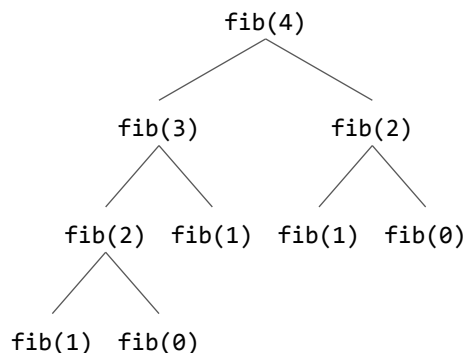
How many times did we call `fib()` to find the fourth Fibonacci number? 9

Do you see a problem with this? What if we tried larger numbers, like 50?

- The number of calls increases exponentially for the initial value of n . This means that we would have to make about 10 billion method calls to calculate `fib(50)`!

How would you rewrite `fib()` to be more efficient, either still as a recursive method or iteratively?

- You need to keep track of the previous two Fibonacci values while computing the next



The Fibonacci Sequence

Here is an example iterative solution:

```
public static long fib(int n) {  
    if (n <= 0) {  
        return 0;  
    }  
  
    long previous = 0;    // at the start, previous = F_0  
    long current = 1;     // at the start, current = F_1  
  
    for (int i = 2; i <= n; i++) {  
        long tmp = previous + current;  
        previous = current;  
        current = tmp;  
    }  
  
    return current;  
}
```

The Fibonacci Sequence

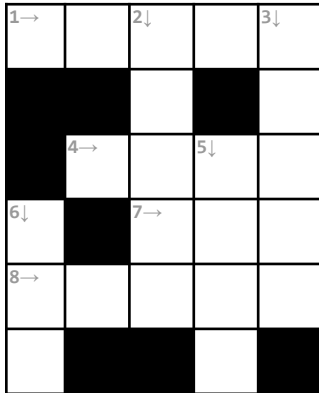
Here is an example recursive solution:

```
public static long fib(int n) {  
    if (n == 0) {  
        return 0;  
    }  
  
    return fibHelper(n, 1, 0);  
}  
  
public static long fibHelper(int n, long curr, long prev) {  
    if (n == 1) {  
        return curr;  
    }  
  
    return fibHelper(n - 1, curr + prev, curr);  
}
```

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

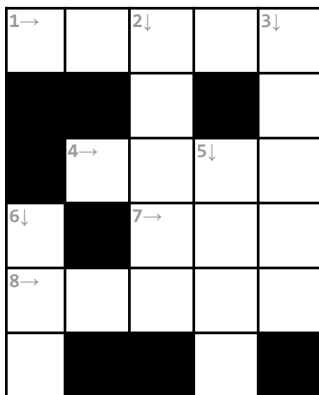
*aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie*



Recursive Backtracking

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*aft
ale
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hoses
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laser
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line
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sheet
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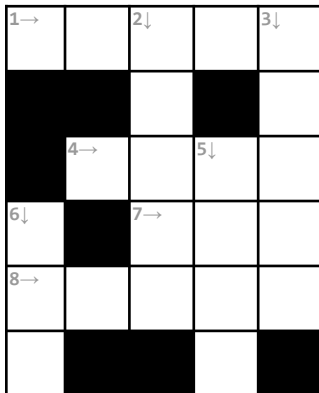


```
boolean findSolution(n, other params) {  
    if (we found a solution) {  
        displaySolution();  
        return true;  
    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params)) {  
                return true;  
            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

Recursive Backtracking

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        }  
    }  
    return false;  
}
```

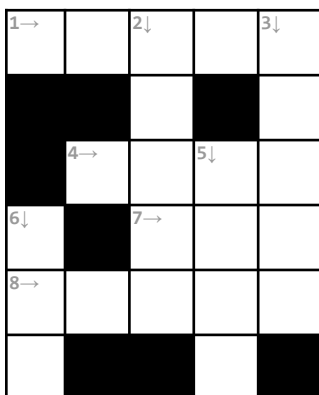
This call to `findSolution()` is trying to place a value in position `n`

In this problem, `n` identifies the slot we're trying to put a word in, e.g. 5 down

Recursive Backtracking

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            removeValue(val);  
        }  
    }  
    return false;  
}
```

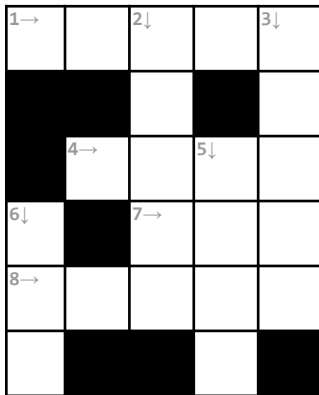
The base case is when a solution is found

In this problem, a solution is found when all 8 slots are filled

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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            }
            removeValue(val);
        }
    }

    return false;
}
```

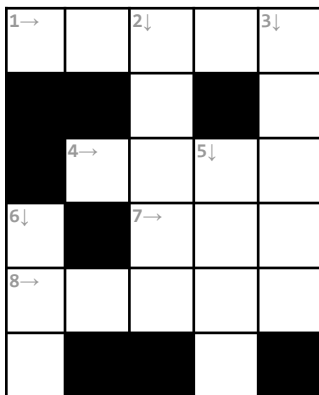
For all possible values that can be put in positions

In this problem, the words from the word list are the values

Recursive Backtracking

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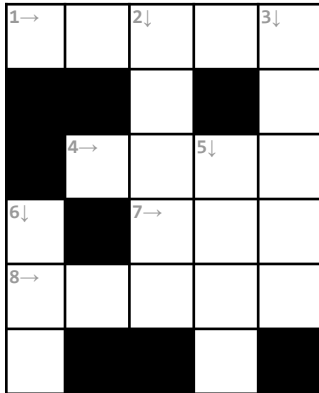
isValid() checks whether the current potential value can be placed in the position

In this problem, what are the constraints for whether a word can be placed in a slot?

Recursive Backtracking

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            }  
            removeValue(val);  
        }  
    }  
    return false;  
}
```

isValid() checks whether the current potential value can be placed in the position

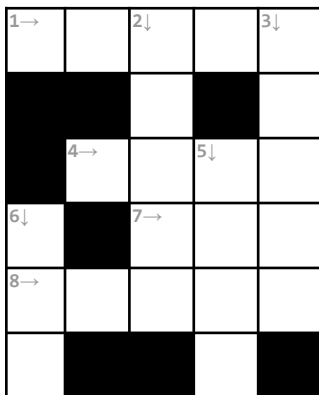
In this problem, what are the constraints for whether a word can be placed in a slot?

1. Intersecting letters match
2. Must fit in slot (length)
3. Word not yet used

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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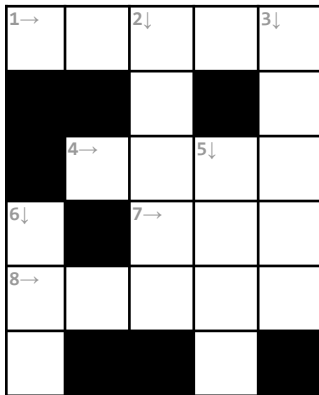
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        return true;  
    }  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params)) {  
                return true;  
            }  
            removeValue(val);  
        }  
    }  
    return false;  
}
```

Add value to the board

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

aft
ale
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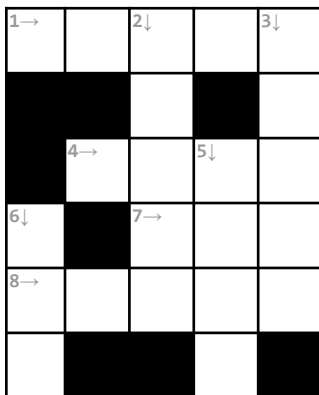
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            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

Make a recursive call to move on to next position (n+1) to try to continue to fill board

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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                return true;  
            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

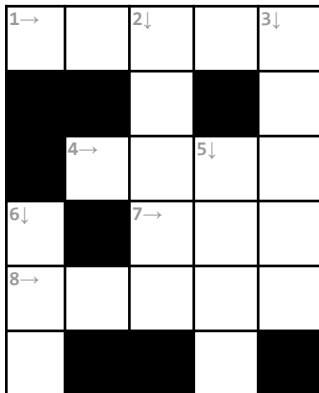
Make a recursive call to move on to next position (n+1) to try to continue to fill board

There are two possibilities of what happens next

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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            }
            removeValue(val);
        }
    }

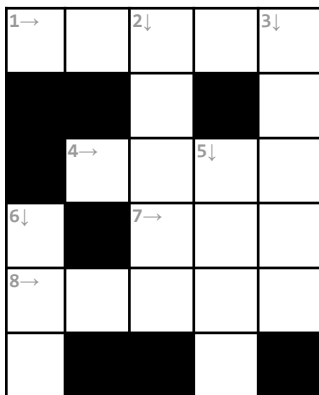
    return false;
}
```

(1) If we continue to place values in positions until we find a solution, we return true in the base case and return true all the way back through the recursive call stack

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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            }
            removeValue(val);
        }
    }

    return false;
}
```

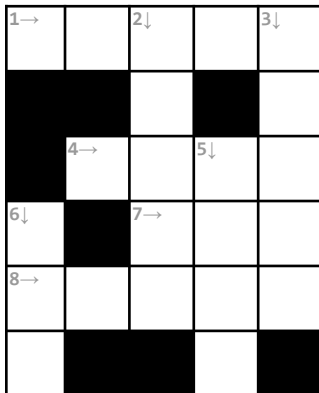
(2) If we reach a position n that we cannot fill -- because none of the values are valid for that position -- the for loop is exhausted

We then return false. We must backtrack!

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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    }

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}
```

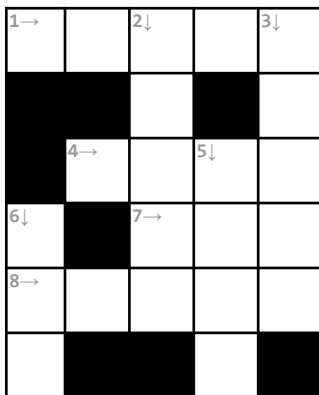
(2) When we backtrack, the recursive call returns false, indicating that placing that value on the board resulted in an unsolvable state

We must remove the value from the board and continue in the for loop to try to place other values

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

aft
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```
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        return true;
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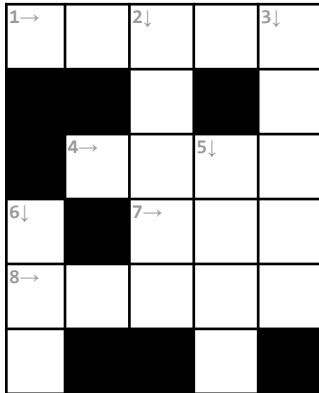
    for (val = first to last) {
        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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```

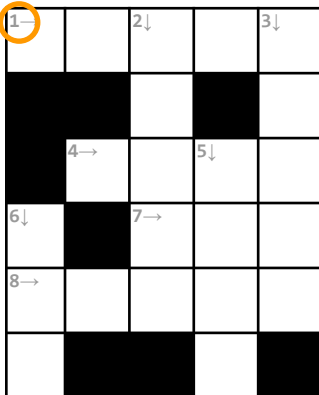
Notice that, in order to do a sample run, we need to assign an arbitrary order in which to assign the spaces in the crossword puzzle.

We use the following order for this run: 1ACROSS, 2DOWN, 3DOWN, 4ACROSS, 7ACROSS, 5DOWN, 8ACROSS, 6DOWN.

Recursive Backtracking

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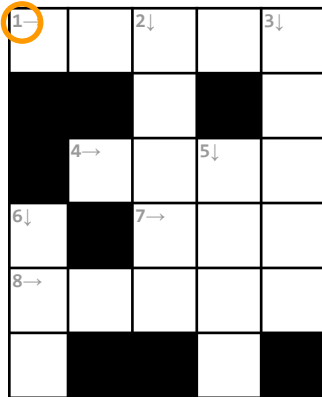
    return false;
}
```

1 across

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {
    if (we found a solution) {
        displaySolution();
        return true;
    }

    for (val = first to last) {
        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

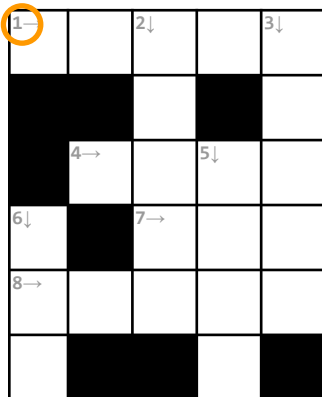
    return false;
}
```

1 across

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
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        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

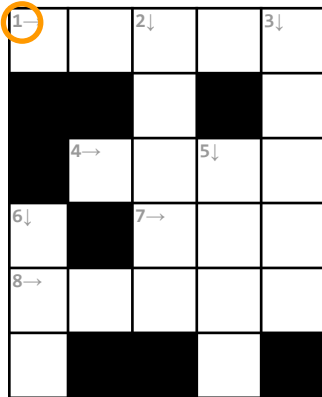
1 across

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {  
    if (we found a solution) {  
        displaySolution();  
        return true;  
    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params)) {  
                return true;  
            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

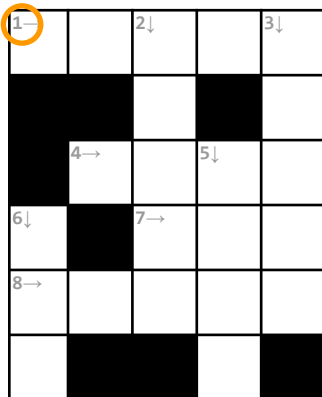
1 across val "aft"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {  
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    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params)) {  
                return true;  
            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

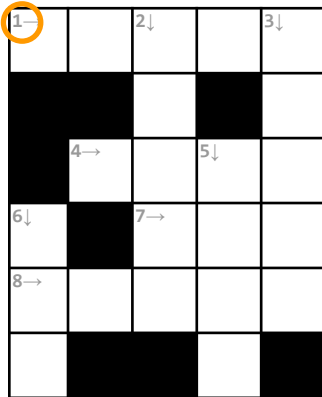
1 across val "ale"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {  
    if (we found a solution) {  
        displaySolution();  
        return true;  
    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params)) {  
                return true;  
            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

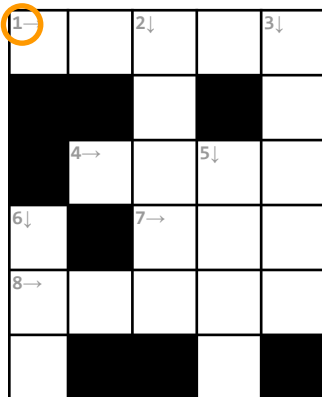
1 across val "eel"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {  
    if (we found a solution) {  
        displaySolution();  
        return true;  
    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params)) {  
                return true;  
            }  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

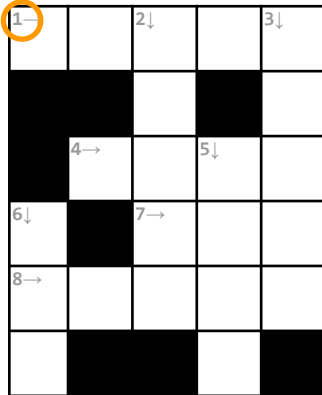
1 across val "heel"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
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    }

    for (val = first to last) {
        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

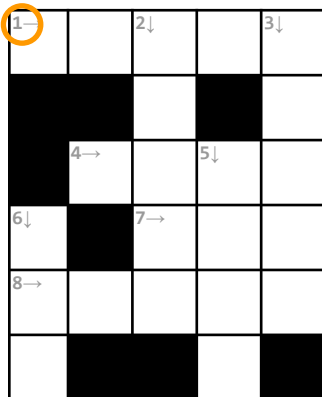
1 across val "hike"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
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    }

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        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

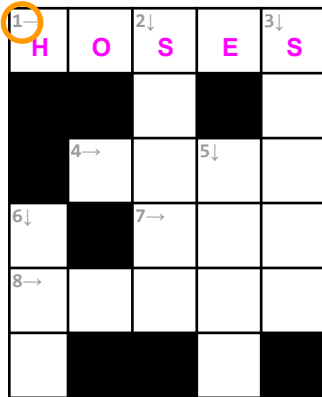
1 across val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {
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            }
            removeValue(val);
        }
    }

    return false;
}
```

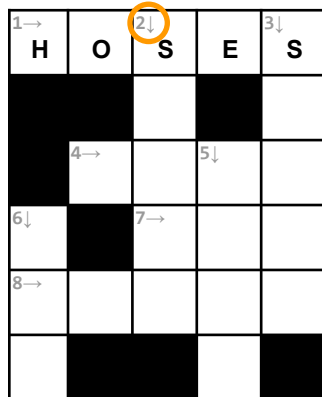
1 across val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
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            }
            removeValue(val);
        }
    }

    return false;
}
```

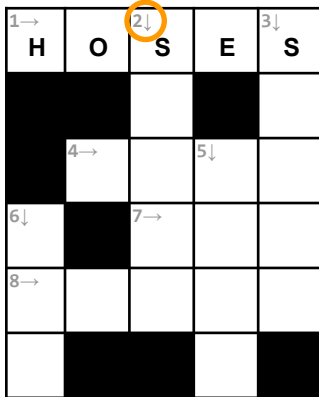
2 down
1 across val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
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laser
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            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

2 down

1 across

val

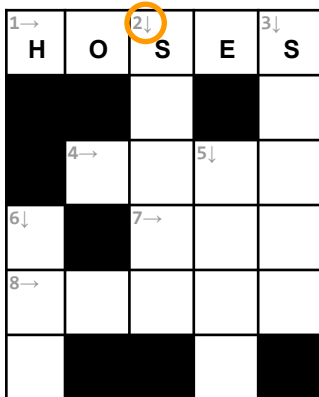
"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
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eel
heel
hike
hoses
keel
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            if (findSolution(n + 1, other params)) {
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            }
            removeValue(val);
        }
    }

    return false;
}
```

2 down

1 across

val

"aft"

val

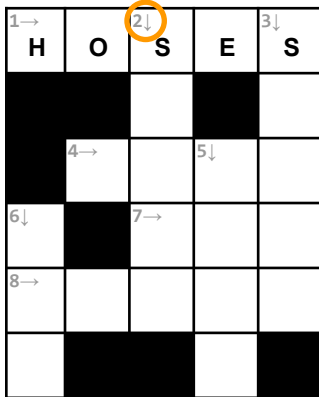
"hoses"

Recursive Backtracking

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1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
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            }
            removeValue(val);
        }
    }

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}
```

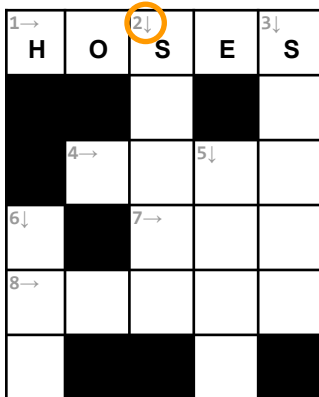
2 down	val	"ale"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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2. Must fit in slot (length)
3. Nothing Given

aft
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eel
heel
hike
hoses
keel
laser
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steer
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            }
            removeValue(val);
        }
    }

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}
```

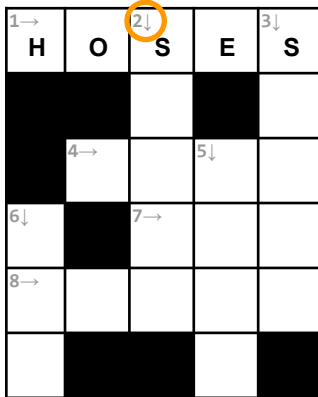
2 down	val	"eel"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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2. Must fit in slot (length)
3. Nothing given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
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            if (findSolution(n + 1, other params)) {
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            }
            removeValue(val);
        }
    }

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}
```

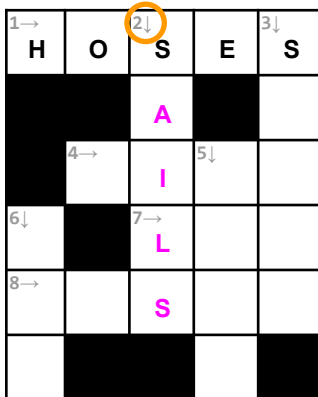
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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2. Must fit in slot (length)
3. Nothing given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
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steer
tie



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            removeValue(val);
        }
    }

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}
```

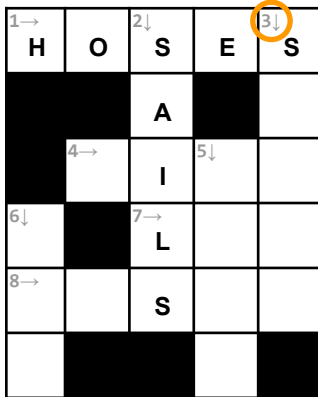
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



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            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

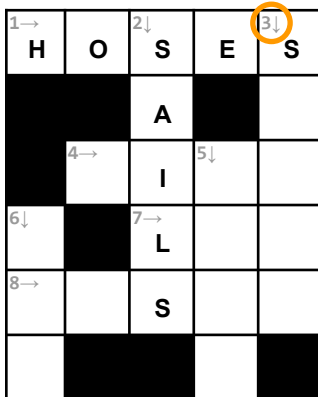
3 down	
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
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            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

3 down	val "aft"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			
	4→			I	5↓		
6↓			7→	L			
8→			S				

```
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            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

3 down	val	"ale"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Working backwards

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→			I	5↓		E
6↓			7→	L			E
8→			S				T

```
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            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
 ale
 eel
 heel
 hike
 hoses
 keel
 laser
 lee
 line
 sails
 sheet
 steer
 tie

1→	H	O	2↓	S	E	3↓	S
				A			H
		4→		I	5↓		E
6↓			7→	L			E
8→			S				T

```

boolean findSolution(n, other params) {
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    for (val = first to last) {
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            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
  
```

4 across	val	
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
 ale
 eel
 heel
 hike
 hoses
 keel
 laser
 lee
 line
 sails
 sheet
 steer
 tie

1→	H	O	2↓	S	E	3↓	S
				A			H
		4→		I	5↓		E
6↓			7→	L			E
8→			S				T

```

boolean findSolution(n, other params) {
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    for (val = first to last) {
        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
  
```

4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {
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        displaySolution();
        return true;
    }

    for (val = first to last) {
        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params))
                return true;
            removeValue(val);
        }
    }

    return false;
}
```

7 across	val	
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
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laser
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        displaySolution();
        return true;
    }

    for (val = first to last) {
        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params))
                return true;
            removeValue(val);
        }
    }

    return false;
}
```

7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {  
    if (we found a solution) {  
        displaySolution();  
        return true;  
    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params))  
                return true;  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

5 down	val	"keel"
7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



```
boolean findSolution(n, other params) {  
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    }  
  
    for (val = first to last) {  
        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params))  
                return true;  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

8 across		
5 down	val	"keel"
7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→	H	I	5↓	K	E	
6↓			7→	L	E	E	
8→			S	E	T		
				L			

No words are valid! Must backtrack

```
boolean findSolution(n, other params) {
    if (we found a solution) {
        displaySolution();
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    }

    for (val = first to last) {
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                return true;
            removeValue(val);
        }
    }

    return false;
}
```

8 across	
5 down	val "keel"
7 across	val "lee"
4 across	val "hike"
3 down	val "sheet"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

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2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→	H	I	5↓	K	E	
6↓			7→	L	E	E	
8→			S	E	T		
				L			

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    }

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}
```

5 down	val "keel"
7 across	val "lee"
4 across	val "hike"
3 down	val "sheet"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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- 2. Must fit in slot (length)
- 3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→	H	I	5↓	K	E	
6↓			7→	L	E	E	
8→			S				T

```
boolean findSolution(n, other params) {
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                return true;
            removeValue(val);
        }
    }

    return false;
}
```

5 down		
7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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aft
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heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→	H	I	5↓	K	E	
6↓			7→	L	E	E	
8→			S				T

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}
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5 down		
7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

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aft
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eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→	H	I	5↓	K	E	
6↓			7→	L	E	E	
8→			S			T	

No words are valid! Must backtrack

```
boolean findSolution(n, other params) {
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        return true;
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    for (val = first to last) {
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    }

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}
```

5 down		
7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

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eel
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keel
laser
lee
line
sails
sheet
steer
tie

1→	H	O	2↓	S	E	3↓	S
				A			H
	4→	H	I	5↓	K	E	
6↓			7→	L		E	
8→			S			T	

```
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            removeValue(val);
        }
    }

    return false;
}
```

7 across	val	"lee"
4 across	val	"hike"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

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3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



No words are valid! Must backtrack

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        if (isValid(val)) {
            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

7 across

4 across val "hike"

3 down val "sheet"

2 down val "sails"

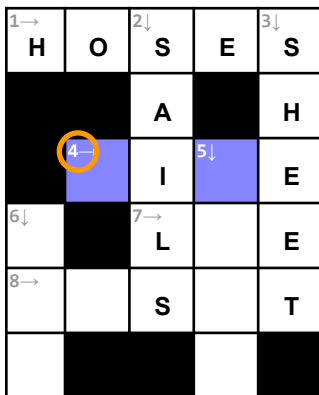
1 across val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
hike
hoses
keel
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line
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            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

4 across val "hike"

3 down val "sheet"

2 down val "sails"

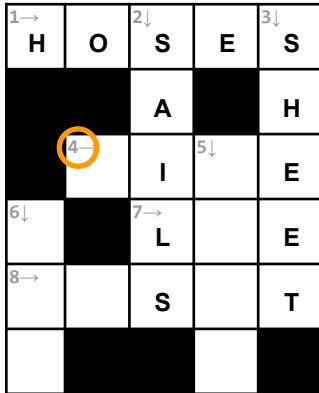
1 across val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
3. Nothing Given

aft
ale
eel
heel
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hoses
keel
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lee
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steer
tie



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            applyValue(val);
            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

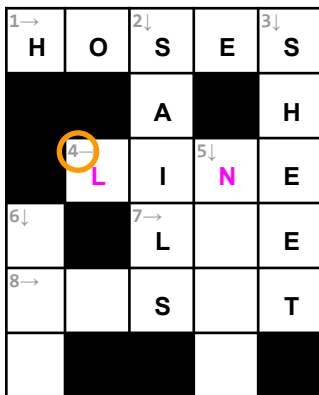
4 across	
3 down	val "sheet"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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            if (findSolution(n + 1, other params)) {
                return true;
            }
            removeValue(val);
        }
    }

    return false;
}
```

4 across	val "line"
3 down	val "sheet"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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ale
eel
heel
hike
hoses
keel
laser
lee
line
sails
sheet
steer
tie



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            if (findSolution(n + 1, other params))
                return true;
            removeValue(val);
        }
    }

    return false;
}
```

7 across	
4 across	val "line"
3 down	val "sheet"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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hoses
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lee
line
sails
sheet
steer
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            applyValue(val);
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                return true;
            removeValue(val);
        }
    }

    return false;
}
```

Notice that, since this is a fresh stack frame for $n == 7$, we start the loop over from the beginning again!

7 across	
4 across	val "line"
3 down	val "sheet"
2 down	val "sails"
1 across	val "hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

1. Intersecting letters match
2. Must fit in slot (length)
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aft
ale
eel
heel
hike
hoses
keel
laser
lee
line
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sheet
steer
tie



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    }  
  
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}
```

Notice that, since this is a fresh stack frame for $n == 7$, we start the loop over from the beginning again!

7 across	val	"lee"
4 across	val	"line"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"

Recursive Backtracking

We consider another problem that can be solved with recursive backtracking. Given a list of words that are to be filled into a crossword puzzle, how do we find a solution that satisfies the rules of the conventional crossword?

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aft
ale
eel
heel
hike
hoses
keel
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sheet
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        if (isValid(val)) {  
            applyValue(val);  
            if (findSolution(n + 1, other params))  
                return true;  
            removeValue(val);  
        }  
    }  
  
    return false;  
}
```

And so on...

7 across	val	"lee"
4 across	val	"line"
3 down	val	"sheet"
2 down	val	"sails"
1 across	val	"hoses"