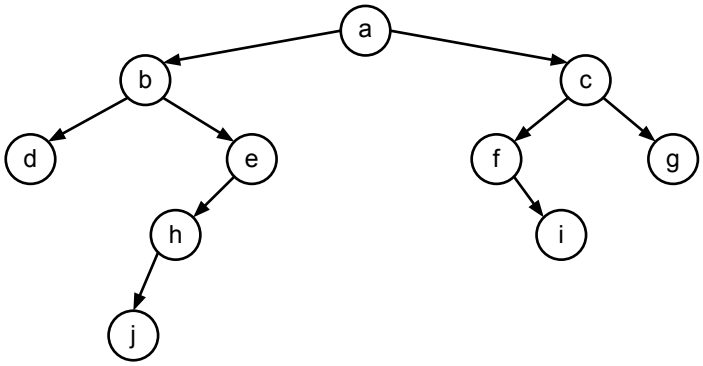
Binary trees

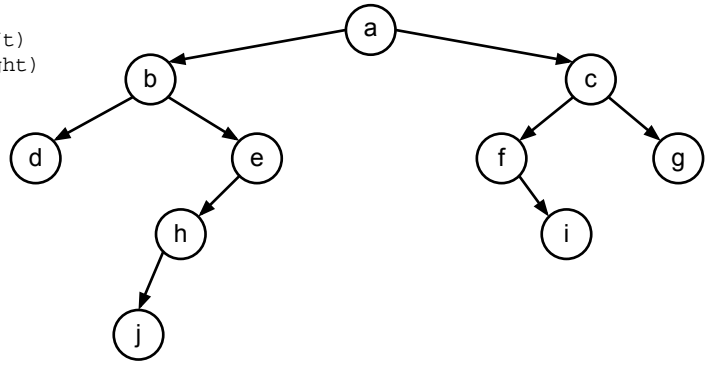
```
public class LinkedTree {
  private class Node {
    private int key;
    private LLList data;
    private Node left;
    private Node right;
    ...
  }

  private Node root;

  ...
}
```



1. What are the ancestors of node *h*?
2. What are the descendants of node *c*?
3. What is the depth of node *i*?
4. What is the height of this tree?

Binary tree traversals

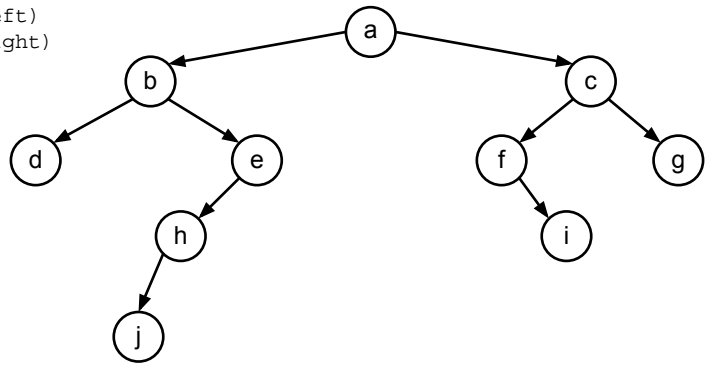
```
preorder(node) {
  print(node.key)
  preorder(node.left)
  preorder(node.right)
}
```



- We want to traverse the tree and print the key of a node when visited
- In what order would the keys be printed if a **pre-order traversal** is made?

Binary tree traversals

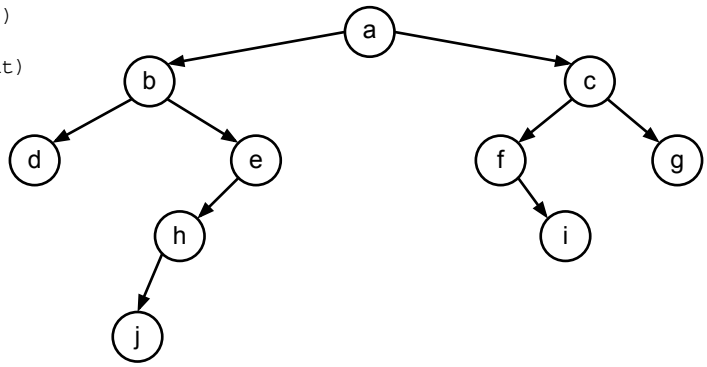
```
postorder(node) {
  postorder(node.left)
  postorder(node.right)
  print(node.key)
}
```



- We want to traverse the tree and print the key of a node when visited
- In what order would the keys be printed if a **post-order traversal** is made?

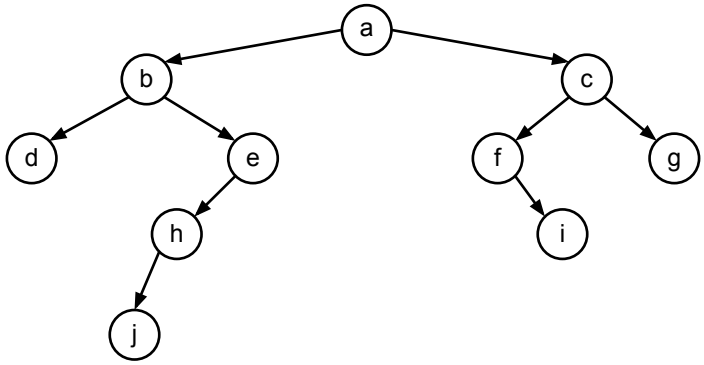
Binary tree traversals

```
inorder(node) {
  inorder(node.left)
  print(node.key)
  inorder(node.right)
}
```



- We want to traverse the tree and print the key of a node when visited
- In what order would the keys be printed if an **in-order traversal** is made?

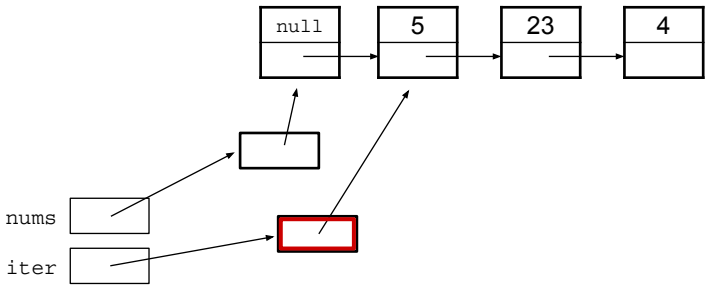
Binary tree traversals



- We want to traverse the tree and print the key of a node when visited
- In what order would the keys be printed if a **level-order traversal** is made?

Iterators for linked lists

```
public class LLList ... {
  private class Node {
    private Object item;
    private Node next;
  }

  private class LLListIterator ... {
    private Node nextNode;

    public LLListIterator() {
      nextNode = head.next;
    }

    public boolean hasNext() {
      return nextNode != null;
    }

    public Object next() {
      Object item = nextNode.item;
      nextNode = nextNode.next;
      return item;
    }
  }
}
```




```
int[] numsArray = {5, 23, 4};
LLList nums = new LLList(numsArray);

ListIterator iter = nums.iterator();
System.out.println("printing all items...");

while (iter.hasNext()) {
  Object item = iter.next();
  System.out.println(item);
}
```

Iterators for binary trees

- Just like linked list iterators, binary tree iterators give consecutive access to values in nodes
- Binary tree iterators should satisfy the following interface:

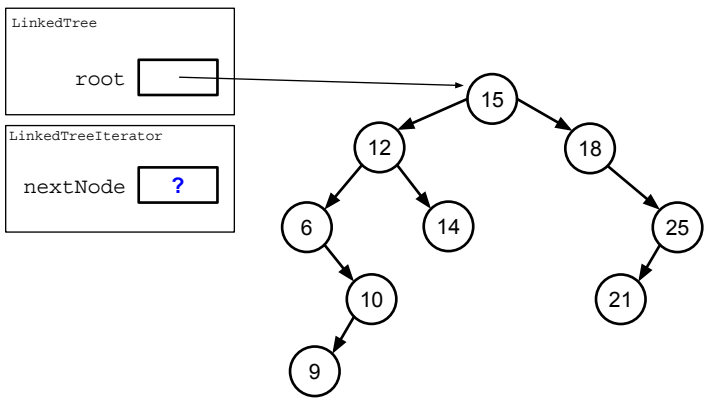
```
public interface LinkedTreeIterator {
  boolean hasNext();
  int next();          // assume tree stores integers
}
```

- Depending on the traversal, we will write a class that implements `LinkedTreeIterator` and express the logic of the traversal **in three places**: constructor, `hasNext()` and `next()`
- Like `LLList`, we implement the iterator as a private inner class

Iterators for binary trees

- Start by copying `nextNode` and the `hasNext()` code from the linked list iterator
- When an iterator is constructed, what node should `nextNode` point to?
  - In other words, which node is visited first in a pre-order traversal?

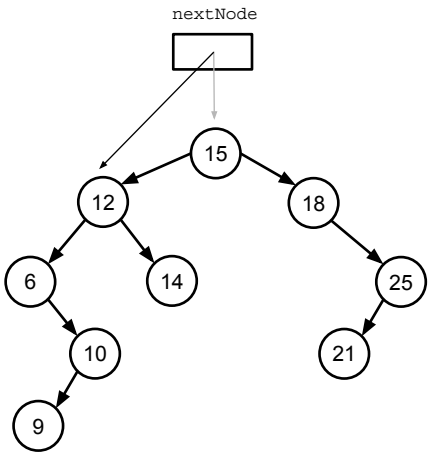
```
public class LinkedTree ... {
  private Node root;

  private class PreorderIterator ... {
    private Node nextNode;

    public PreorderIterator() {
      nextNode = ???
    }

    public boolean hasNext() {
      return nextNode != null;
    }

    public int next() {
      ???
    }
  }
}
```



Iterators for binary trees: the first call to `next()`

- For the tree on the right, the first two keys in the pre-order traversal are 15, then 12
- After an iterator is constructed, the first call to `next()` would need to create a variable for the key of the 15 node, then make `nextNode` point to the 12 node

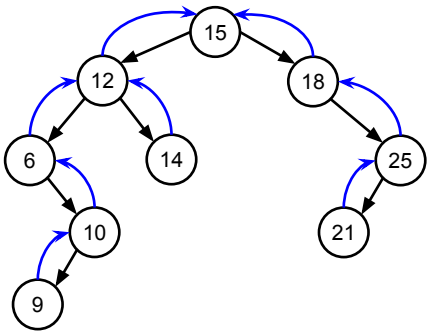
```
private class PreorderIterator ... {
  private Node nextNode;

  public int next() {









  }
}
```



Iterators for binary trees: `parent` references

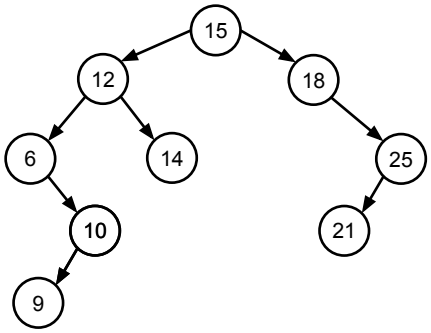
- To enable our binary tree iterator to reach all nodes starting from any node, we can add a `parent` reference to the inner `Node` class
- The methods of the `LinkedTree` class must be changed to properly update the `parent` reference (e.g., when inserting a node)

```
private class Node {
  private int key;
  private LLList data;
  private Node left;
  private Node right;
  private Node parent;
  ...
}
```



Iterators for binary trees: improving `next()`

```
public int next() {
  int toReturn = nextNode.key;

  if (nextNode.left != null) {
    nextNode = nextNode.left;
  } else if (nextNode.right != null) {
    nextNode = nextNode.right;
  } else {









  }

  return toReturn;
}
```



Huffman encoding

- We are given a document where all characters are drawn from a set of 6 characters, with the frequencies shown here
- Let's create a Huffman tree from this table of frequencies and then use it to decode a binary string
- To create a Huffman tree, create nodes for each character, then, keeping the nodes in sorted order, repeatedly combining the two lowest-frequency nodes into a subtree

character	frequency
e	45
a	33
r	20
i	18
n	15
d	10

Huffman encoding

character	frequency
e	45
a	33
r	20
i	18
n	15
d	10

Huffman encoding

- Let's use the tree to decode the following binary string:

```
00101000100101
```